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The convolution operator in the smooth functions spaces

The abstract of dissertation is presented for the scientific degree of the PhD at specialty of 6D060100- Mathematics

The structure and scope of the thesis. The dissertation work consists of an introduction, two sections (each section consists of paragraphs), a conclusion and a list of used literature. The total number of pages is 71.

The number of illustrations, tables, used literary sources. The number of sources used is 52.

Keywords. Convolution operator, anisotropic Lorentz spaces, anisotropic Sobolev spaces, anisotropic Nikol'skii-Besov spaces, anisotropic Triebel-Lizorkin spaces.

Relevance of the topic.

The boundedness conditions for the convolution operator are well studied in the Lebesgue and Lorentz spaces. This issue has not been sufficiently studied for smooth functions spaces, especially for anisotropic spaces.

In this dissertation, we study the conditions for the boundedness of the convolution operator in smooth functions spaces as anisotropic Triebel-Lizorkin, Sobolev, Nikol'skii-Besov spaces. By anisotropic space, we mean the space of functions that have different differential and metric properties in each variable.

The relevance of the topic of the dissertation is explained by the fact that at the moment a large cycle of problems of estimates of the convolution operator norm for isotropic Sobolev and Nikol'skii-Besov spaces with a dominant mixed derivative has been studied. The results of these studies are actively used in solving boundary value problems of mathematical physics equations, as well as in the compression and restoration of information. In these studies, the anisotropy condition in the smoothness characteristic and isotropy in the metric characteristic of the functions are imposed on the function spaces.

The aim of the research is to obtain a boundedness condition for the convolution operator for spaces of smooth functions: anisotropic Sobolev, Nikol'skii-Besov, Triebel-Lizorkin spaces.

Research objectives:

1. Investigate new anisotropic spaces of smooth functions.
2. Investigation of conditions for boundedness of the convolution operator in anisotropic Lorentz spaces.
3. Investigation of the interpolation properties of anisotropic spaces.
4. Investigation of conditions for boundedness of the convolution operator in anisotropic Nikol'skii-Besov spaces.
5. Investigation of conditions for boundedness of the convolution operator in anisotropic Lizorkin-Triebel spaces.

Research methods. The dissertation uses the methods of the theory of interpolation of anisotropic spaces, and also uses the methods of the function spaces

theory, methods of functional analysis, methods and inequalities of harmonic analysis.

The objects of research are the convolution operator, anisotropic Lorentz spaces, anisotropic Sobolev, Nikol'skii-Besov, Triebel-Lizorkin spaces with a dominant mixed derivative.

Scientific novelty and practical value of the work.

1. They were investigated and obtained Young-O'Neill type inequalities in anisotropic Lorentz spaces.

2. For Sobolev spaces with a dominant mixed derivative, an analogue of Young's inequality is obtained, namely, relations of the form $W_{\mathbf{p}}^{\gamma} * W_{\mathbf{r}}^{\beta} \hookrightarrow W_{\mathbf{q}}^{\alpha}$ are proved under the corresponding conditions on the parameters.

3. It was obtained an analogue of O'Neill's theorem for the scale of Nikol'skii-Besov spaces, where $\boldsymbol{\alpha}$, $\mathbf{p}$, $\mathbf{q}$ are vector parameters. These results complement the results of Batyrov and Burenkov, where similar problems were considered in isotropic Nikol'skii-Besov spaces, that is, in spaces where parameters are scalars.

4. It was obtained an analogue of O'Neill's theorem for the scale of Triebel-Lizorkin spaces, that is, relations of the form $F_{\mathbf{r},\mu}^{\beta,\eta} * F_{\mathbf{h},\nu}^{\gamma,\xi} \hookrightarrow F_{\mathbf{p},\tau}^{\alpha,q}$, with the corresponding relations of the vector parameters.

5. Necessary and sufficient conditions are obtained for the embeddings $B_{\mathbf{r},\mu}^{\beta,\eta} * B_{\mathbf{h},\nu}^{\gamma,\xi} \hookrightarrow B_{\mathbf{p},\tau}^{\alpha,q}$ in terms of the corresponding vector parameters $\boldsymbol{\alpha}$, $\mathbf{p}$, $\mathbf{q}$, $\boldsymbol{\tau}$, $\mathbf{r}$, $\boldsymbol{\mu}$, $\boldsymbol{\beta}$, $\boldsymbol{\eta}$, $\mathbf{h}$, $\boldsymbol{\nu}$, $\boldsymbol{\gamma}$, $\boldsymbol{\xi}$. In particular, the results proved above imply the classical O'Neill inequality. The obtained criterion generalizes the results of Burenkov and Batyrov, who considered this problem in Nikol'skii-Besov spaces with scalar parameters.

The work is theoretical, their scientific significance is due to the use of deep, modern mathematical results. The used methods and the obtained results can be applied to further study the conditions for boundedness of the convolution operator, in embedding theorems for function spaces. In addition, the results can be used in solving boundary value problems for equations of mathematical physics and in reading special courses on the theory of harmonic analysis.

Provisions for Defense.

1. The O'Neill type inequalities for anisotropic Lorentz spaces.

2. An example showing the accuracy of the result of O'Neill-type inequalities for anisotropic Lorentz spaces.

3. Estimates for the convolution operator norm in anisotropic Besov spaces with dominant mixed derivative.

4. An analogue of O'Neill's theorem for the scale of Triebel-Lizorkin spaces.

5. Necessary and sufficient conditions for investments $B_{\mathbf{r},\mu}^{\beta,\eta} * B_{\mathbf{h},\nu}^{\gamma,\xi} \hookrightarrow B_{\mathbf{p},\tau}^{\alpha,q}$.

Publications. The main results of the dissertation were published in 11 papers. Of these, 3 articles in publications recommended by the authorized body, 1 article in the journal from the Scopus database with a CiteScore percentile of no less than 25, also participated in 7 international scientific conferences.